## Simple description of neutrinos in SU(5)

## Noriyuki Oshimo

Institute of Humanities and Sciences and Department of Physics

Ochanomizu University, Tokyo, 112-8610, Japan

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## Abstract

We show that experimental results for the masses and mixing of the neutrinos can be understood naturally by a simple grand unification model of SU(5) coupled to N=1 supergravity. No right-handed neutrinos are included. The left-handed neutrinos receive Majorana masses through the couplings with a Higgs boson of symmetric 15 representation. Introducing  $\overline{\bf 45}$  representation is optional for describing the masses of down-type quarks and charged leptons.

Accumulating experimental results for solar [1] and atmospheric [2] neutrino oscillations suggest that the neutrinos have extremely small but non-vanishing masses. Some extension of the standard model (SM) must be necessary. In addition, large angles for the generation mixing of leptons have been observed, contrary to the small angles for the quark generation mixing. Combined with experimental results for the neutrino oscillations by nuclear reactors [3], detailed information on the lepton generation mixing has been obtained. How should the SM be extended?

For physics beyond the SM, irrespectively of the problem on the neutrinos, grand unified theories (GUTs) coupled to supersymmetry are very plausible from various theoretical viewpoints. On the other hand, one of the most popular scenarios for incorporating the neutrino masses is to introduce right-handed neutrinos with large Majorana masses [4]. The huge mass differences between the neutrinos and the charged leptons could then be explained naturally. This scenario is embodied in the GUT models whose gauge group is SO(10) or a larger one. Unfortunately, the minimal GUT group of SU(5) does not contain a representation for right-handed neutrinos, unless ad hoc SU(5)-singlet fields are introduced. Therefore, SO(10) GUT models might be now considered as most promising for the extension of the SM.

One obstacle, however, confronts the GUT models based on SO(10) or a larger group. The generation mixing for the quarks and that for the leptons are described respectively by the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the Maki-Nakagawa-Sakata (MNS) matrix. These matrices are generically related to each other in those models. The observed difference between the CKM matrix and the MNS matrix is not trivially understood. Various approaches thus have been tried to explain the difference [5], often contrived schemes being invoked. The SO(10) GUT models also suffer from high dimensions of Higgs boson representations. It is a complicated procedure to set, among their various fields decomposed under  $SU(3) \times SU(2) \times U(1)$ , two with quantum numbers (1,2,-1/2) and (1,2,1/2) light while the others heavy for having consistent phenomenology.

In this letter we present a simple GUT model which can describe naturally the masses and mixing of the neutrinos. The gauge group is the minimal SU(5) and supersymmetry is imposed by coupling the model to N=1 supergravity. The right-handed neutrinos are not included. Within the framework of SU(2)×U(1) electroweak theory, the left-handed neutrinos could have small Majorana masses without fine-tuning, if an SU(2)-triplet Higgs

TABLE I: The SU(5) quantum numbers of the superfields. The suffixes a, b, and c denote the group indices. The generation index is represented by i = (1 - 3).

$\Phi^a_b$	$T^{ab}$	$\overline{T}_{ab}$	$H^a$	$\overline{H}_a$
24	15	$\overline{15}$	5	$\overline{5}$
$\Psi_i^{ab}$	$\overline{\Psi}_{ia}$	$(U_c^{ab})$	$(\overline{U}_{ab}^c)$	
10	<u>5</u>	45	$\overline{45}$	

boson is appropriately incorporated [6]. This mechanism is naturally embedded in our model. The CKM matrix and the MNS matrix become independent of each other. Their observed difference is merely due to different values for free parameters, as so are the mass differences among the quarks and the leptons.

The model consists of Higgs superfields  $\Phi$ , T,  $\overline{T}$ , H, and  $\overline{H}$ , and matter superfields  $\Psi_i$  and  $\overline{\Psi}_i$ , with i being the generation index. Their quantum numbers are shown in Table I. In addition to the particle contents of the minimal SU(5) model, superfields of symmetric 15 and  $\overline{\bf 15}$  representations are introduced. This 15 representation can couple to  $\overline{\bf 5} \times \overline{\bf 5}$  of matters, leading to Majorana masses for the left-handed neutrinos at the electroweak energy scale. The complex conjugate  $\overline{\bf 15}$  representation is needed in order to render T very heavy, as well as anomaly free. These particle contents alone, however, may encounter a difficulty in describing masses of some down-type quarks or charged leptons, which is a common problem for ordinary minimal SU(5) models [7]. One of its solutions is to introduce an additional Higgs superfield  $\overline{U}$  of  $\overline{\bf 45}$  representation, which couples to the matters  ${\bf 10} \times \overline{\bf 5}$ . In this case, the complex conjugate  ${\bf 45}$  representation is also included for having a mass term and anomaly cancellation. This option is discussed afterward.

The superpotential of the model is given by the sum of two sectors,

$$W_{H} = M_{H}\overline{H}H + M_{T}\operatorname{Tr}[\overline{T}T] + \frac{1}{2}M_{\Phi}\operatorname{Tr}[\Phi^{2}]$$

$$+ \lambda_{H\Phi}\overline{H}\Phi H + \lambda_{T\Phi}\operatorname{Tr}[\overline{T}\Phi T] + \frac{1}{3}\lambda_{\Phi}\operatorname{Tr}[\Phi^{3}]$$

$$+ \lambda_{HT}H\overline{T}H + \overline{\lambda}_{HT}\overline{H}T\overline{H}, \qquad (1)$$

$$W_M = \Gamma^u_{ij} \epsilon H \Psi_i \Psi_j + \Gamma^{de}_{ij} \overline{H} \Psi_i \overline{\Psi}_j + \Gamma^{\nu}_{ij} \overline{\Psi}_i T \overline{\Psi}_j, \tag{2}$$

where  $\epsilon$  denotes the totally antisymmetric tensor of rank five. Contraction of SU(5) group indices is understood. Involved are all the renormalizable terms consistent with SU(5) and R

parity. The mass parameters  $M_H$ ,  $M_T$ , and  $M_{\Phi}$  have values of the order of the GUT energy scale  $M_X$ , which is given typically by  $M_X \sim 10^{16}$  GeV. The superpotential  $W_H$  contains only Higgs superfields and determines the vacuum. The vacuum expectation value (VEV) of  $\Phi$  is responsible for breakdown of SU(5), and those of H and  $\overline{H}$  are to break SU(2)×U(1). The VEV of T induces masses of the left-handed neutrinos. The masses of the quarks and leptons are generated by the superpotential  $W_M$ . The coefficients  $\Gamma^u$  and  $\Gamma^{\nu}$  are symmetric for generation indices.

The SU(5) gauge symmetry is broken at the vacuum. Although the superpotential  $W_H$  yields three degenerated vacua under global supersymmetry, this degeneracy is lifted by N=1 supergravity [8]. The scalar component of the adjoint representation  $\Phi$  could have a VEV  $\langle \Phi \rangle = \text{diag}(1,1,1,-3/2,-3/2)v_{\Phi}$ , where  $v_{\Phi}$  is given by  $v_{\Phi} \simeq 2\text{Re}(M_{\Phi})/\text{Re}(\lambda_{\Phi})$  and related to the X and Y boson masses as  $M_X^2 = M_Y^2 = (25/8)g_5^2v_{\Phi}^2$ . The VEVs of the other scalar fields vanish. Then, the gauge symmetry after the breaking of SU(5) is  $\text{SU}(3)\times\text{SU}(2)\times\text{U}(1)$  of the SM.

Below the GUT energy scale, the vacuum is prescribed by the scalar potential which consists of the SU(3)-singlet components for H,  $\overline{H}$ , T,  $\overline{T}$ , and  $\Phi$ . The relevant part of the superpotential  $W_H$  is written as

$$W_{H} = -m_{H}H_{1}\epsilon H_{2} + m_{T}\operatorname{Tr}[\overline{T}T] + \frac{1}{2}m_{\Phi}\operatorname{Tr}[\Phi^{2}]$$

$$+ \lambda_{\phi 1}(\epsilon H_{1})^{T}\Phi H_{2} + \lambda_{\phi 2}\operatorname{Tr}[\overline{T}\Phi T] + \frac{1}{3}\lambda_{\phi 3}\operatorname{Tr}[\Phi^{3}]$$

$$+ \overline{\lambda}(\epsilon H_{1})^{T}T\epsilon H_{1} + \lambda(H_{2})^{T}\overline{T}H_{2}, \tag{3}$$

where  $\epsilon$  stands for the totally antisymmetric tensor of rank two. The SU(2)-doublet components of H and  $\overline{H}$  are expressed by  $H_2$  and  $H_1$  as  $H_2 = (H^4, H^5)$  and  $H_1 = (-\overline{H}_5, \overline{H}_4)$ . The SU(2)-triplet components of  $\Phi$ , T, and  $\overline{T}$  are denoted by the same symbols. At the GUT energy scale, the coefficients have the values  $m_H = M_H - (3/2)\lambda_{H\Phi}v_{\Phi}$ ,  $m_T = M_T - (3/2)\lambda_{T\Phi}v_{\Phi}$ ,  $m_{\Phi} = M_{\Phi} - 3\lambda_{\Phi}v_{\Phi}$ ,  $\lambda_{\phi 1} = \lambda_{H\Phi}$ ,  $\lambda_{\phi 2} = \lambda_{T\Phi}$ ,  $\lambda_{\phi 3} = \lambda_{\Phi}$ ,  $\lambda = \lambda_{HT}$ , and  $\overline{\lambda} = \overline{\lambda}_{HT}$ . We assume that the magnitude of  $m_H$  becomes of the order of the electroweak energy scale  $M_W$  by finetuning or some other mechanism, while the SU(3)-triplet components of H and  $\overline{H}$  remain of the order of  $M_X$ . The mass parameters  $m_T$  and  $m_{\Phi}$  also have a natural magnitude of the order of  $M_X$ .

The superpotential for the quark and lepton masses is written as

$$W_M = \eta_d^{ij} H_1 \epsilon Q^i D^{cj} + \eta_u^{ij} H_2 \epsilon Q^i U^{cj} + \eta_e^{ij} H_1 \epsilon L^i E^{cj}$$

$$+ \frac{1}{2} \kappa^{ij} (\epsilon L^i)^T T \epsilon L^j, \tag{4}$$

where  $Q^i$ ,  $U^{ci}$ ,  $D^{ci}$ ,  $L^i$ , and  $E^{ci}$  stand for the superfields for the quarks and leptons in a self-explanatory notation. There exist possible Majorana mass terms for the left-handed neutrinos. At the GUT energy scale, the coefficients have the values  $\eta_d = -\Gamma^{de}/\sqrt{2}$ ,  $\eta_u = 4\Gamma^u$ ,  $\eta_e = -(\Gamma^{de})^T/\sqrt{2}$ , and  $\kappa = 2\Gamma^{\nu}$ . The physical parameters for the coefficients are given by the CKM matrix  $V_{CKM}$ , the MNS matrix  $V_{MNS}$ , and the diagonalized eigenvalue matrices  $\eta_d^D$ ,  $\eta_u^D$ ,  $\eta_e^D$ , and  $\kappa^D$ .

The scalar potential for  $H_1$  and  $H_2$  is mostly the same as the minimal supersymmetric standard model, since the superfields T,  $\overline{T}$ , and  $\Phi$  have masses of the order of  $M_X$ . In the supersymmetry-soft-breaking Lagrangian, the Higgs bosons  $H_1$  and  $H_2$  have positive masses-squared of the order of  $M_W$  at the GUT energy scale. However, owing to a large value of  $\eta_u^{ij}$  corresponding to the t quark mass, the mass-squared of  $H_2$  is driven negative at the electroweak energy scale through quantum corrections. Non-vanishing VEVs are then induced for those Higgs bosons and  $SU(2)\times U(1)$  is broken down to  $U(1)_{EM}$  [9]. The VEVs of  $H_1$  and  $H_2$  are expressed by  $\langle H_1 \rangle = (v_1/\sqrt{2},0)$  and  $\langle H_2 \rangle = (0,v_2/\sqrt{2})$ , where  $v_1$  and  $v_2$  are related to the W-boson mass as  $M_W^2 = (1/4)g_2^2(v_1^2 + v_2^2)$ .

We first show that the electroweak symmetry breaking induces non-vanishing VEVs for T and  $\overline{T}$ . Assuming that  $\mathrm{U}(1)_{\mathrm{EM}}$  symmetry is not broken, we can put the VEVs of these  $\mathrm{SU}(2)$ -triplet fields at  $\langle T \rangle = \mathrm{diag}(0, v_T/\sqrt{2})$  and  $\langle \overline{T} \rangle = \mathrm{diag}(0, v_{\overline{T}}/\sqrt{2})$ . Since the scalar components of T,  $\overline{T}$ , and  $\Phi$  have large positive masses-squared of the order of  $M_X$  through the F-term scalar potential, the magnitudes of  $v_T$  and  $v_{\overline{T}}$ , as well as the VEV of  $\Phi$ , must be extremely smaller than  $v_1$  and  $v_2$ , albeit non-vanishing. The contribution of  $v_T$  and  $v_{\overline{T}}$  to the VEV of the scalar potential is given by

$$V = \frac{1}{2} |m_T|^2 \left( |v_T|^2 + |v_{\overline{T}}|^2 \right) + \frac{1}{\sqrt{2}} \operatorname{Re} \left( \lambda m_T^* v_2^2 v_T^* \right) + \frac{1}{\sqrt{2}} \operatorname{Re} \left( \overline{\lambda} m_T^* v_1^2 v_{\overline{T}}^* \right),$$
 (5)

where we have neglected the terms whose magnitudes should be smaller than the order of  $(M_W)^4$ . The non-negligible terms all arise from the F-term scalar potential. The values for  $v_T$  and  $v_{\overline{T}}$  become non-vanishing at the vacuum, which are given by

$$|v_T| = \left| \frac{\lambda v_2^2}{\sqrt{2}m_T} \right|, |v_{\overline{T}}| = \left| \frac{\overline{\lambda}v_1^2}{\sqrt{2}m_T} \right|. \tag{6}$$

For  $m_T \sim 10^{14}$  GeV and  $v_1$ ,  $v_2 \sim 10^2$  GeV, the VEVs  $v_T$  and  $v_{\overline{T}}$  are of the order of  $10^{-1}$  eV. The left-handed neutrinos can have the masses which are of the correct order of magnitude. It should be noted that the appropriate value for  $m_T$  is smaller than  $M_X$  by one or two orders of magnitude.

We now turn to the discussion of the CKM and MNS matrices and the eigenvalue matrices for the Higgs coupling coefficients. The values of these matrices depend on the energy scale of physics, which are governed by the renormalization group equations for the coefficient matrices  $\eta_d$ ,  $\eta_u$ ,  $\eta_e$ , and  $\kappa$  appearing in Eq. (4). Between the energy scales  $M_X$  and  $M_W$ , the renormalization group equation of  $\kappa$  is given at one-loop level by

$$\mu \frac{d\kappa}{d\mu} = \frac{1}{16\pi^2} \left\{ \left( 5g_2^2 + \frac{9}{5}g_1^2 + \frac{1}{2} \text{Tr}[\kappa^{\dagger}\kappa] \right) \kappa + \eta_e \eta_e^{\dagger} \kappa + \kappa \eta_e \eta_e^{\dagger} \right\}, \tag{7}$$

where  $g_1$  denotes the normalized U(1) gauge coupling constant. We have taken the generation basis in which  $\eta_e$  is diagonal. For calculations of self-energies and vertex corrections, the contributions from the superfields T and  $\overline{T}$  have been neglected. The renormalization group equations for  $\eta_d$ ,  $\eta_u$ , and  $\eta_e$  are known in the literature. Experimentally measured quantities are the CKM and MNS matrices at the electroweak energy scale. The eigenvalue matrices  $\eta_d^D$ ,  $\eta_u^D$ ,  $\eta_e^D$ , and  $\kappa^D$  at this energy scale could also be determined, provided that the ratios of  $v_1$  to  $v_2$  and to  $v_T$  are given.

The superpotential  $W_M$  in Eq. (2) can accommodate any values for the CKM and MNS matrices. Any masses for the up-type quarks and neutrinos can also be realized. Given the experimentally determined values for  $V_{CKM}$ ,  $V_{MNS}$ ,  $\eta_u^D$ ,  $\eta_d^D$ ,  $\eta_e^D$ , and  $\kappa^D$ , the values which these matrices should have at the GUT energy scale are obtained by the renormalization group equations. For expressing the coefficient matrices, we take the generation basis in which  $\eta_d$  and  $\eta_e$  are diagonal. The required values for  $V_{CKM}$ ,  $V_{MNS}$ ,  $\eta_u^D$ , and  $\kappa^D$  are fulfilled by taking the coefficients as

$$\Gamma^u = \frac{1}{4} (V_{CKM})^T \eta_u^D V_{CKM}, \tag{8}$$

$$\Gamma^{\nu} = \frac{1}{2} (V_{MNS})^T \kappa^D V_{MNS}. \tag{9}$$

These equations can always be satisfied, since the coefficients  $\Gamma^u$ ,  $\Gamma^{de}$ , and  $\Gamma^{\nu}$  are independent mutually.

A problem arises from the masses of down-type quarks and charged leptons. According to  $W_M$  in Eq. (2), the coefficient matrices  $\eta_d$  and  $\eta_e$  should have the same eigenvalues at the GUT energy scale,

$$\Gamma^{de} = -\sqrt{2}\eta_d^D = -\sqrt{2}\eta_e^D. \tag{10}$$

However, the experimental results do not naively lead to these coincidences, except for the b quark and the  $\tau$  lepton. As an example, let us take  $m_d = 4.0$  MeV,  $m_s = 1.1 \times 10^2$  MeV, and  $m_b = 3.1$  GeV for the down-type quark masses, and  $m_e = 4.9 \times 10^{-1}$  MeV,  $m_\mu = 1.0 \times 10^2$  MeV, and  $m_\tau = 1.7$  GeV for the charged lepton masses at the electroweak energy scale. The ratio of the VEVs is put at  $v_2/v_1 = 30$ . Then, at the GUT energy scale, the eigenvalue matrices are given by  $\eta_d^D = (2.1 \times 10^{-4}, 5.8 \times 10^{-3}, 2.0 \times 10^{-1})$  and  $\eta_e^D = (6.0 \times 10^{-5}, 1.2 \times 10^{-2}, 2.2 \times 10^{-1})$ . For the first two generations, there appear differences by a factor of three or two. This difficulty could be relieved if the difference of a few orders of magnitude between  $M_X$  and  $m_T$  is correctly taken into account in the analysis by renormalization group equations. Furthermore, the determination of the light quark masses by experimental results is very difficult. Therefore, the above mentioned discrepancy may not be serious for the present model.

The problem of the down-type quark and charged lepton masses could eventually be solved by including the Higgs superfields U and  $\overline{U}$  of  $\mathbf{45}$  and  $\mathbf{\overline{45}}$ . Then, the superpotentials in Eqs. (1) and (2) have additional terms

$$W_{H} = M_{U}\overline{U}U + \lambda_{U\Phi}\overline{U}\Phi U + \lambda_{HU}H\overline{U}\Phi + \overline{\lambda}_{HU}\overline{H}U\Phi + \lambda_{TU}\overline{U}T\overline{U} + \overline{\lambda}_{TU}U\overline{T}U, \tag{11}$$

$$W_M = \overline{\Gamma}_{ij}^u \epsilon U \Psi_i \Psi_j + \overline{\Gamma}_{ij}^{de} \overline{U} \Psi_i \overline{\Psi}_j. \tag{12}$$

A new source of masses for the down-type quarks and charged leptons is involved, which invalidates the relation in Eq. (10). The up-type quark masses also receive new contributions. The coefficient  $\overline{\Gamma}^u$  is antisymmetric for generation indices. The SU(2)-doublet components of U and  $\overline{U}$  for the Higgs superfields are formed as  $(2\sqrt{6}/3)(-U_5^{45}, -U_4^{54})$  and  $(2\sqrt{6}/3)(\overline{U}_{54}^4, -\overline{U}_{45}^5)$ , with U(1) hypercharges 1/2 and -1/2, respectively. The factor  $2\sqrt{6}/3$  is multiplied for the normalization.

The Higgs superfields  $H_1$  and  $H_2$  in Eq. (4) are composed of the SU(2)-doublet superfields  $H_2^5$ ,  $H_1^{\overline{5}}$ ,  $H_2^{\overline{45}}$ , and  $H_1^{\overline{45}}$  in respectively  $\mathbf{5}$ ,  $\overline{\mathbf{5}}$ ,  $\mathbf{45}$ , and  $\overline{\mathbf{45}}$ . After breakdown of SU(5), they are

mixed by the term

$$W = \left(H_1^{\overline{5}} H_1^{\overline{45}}\right) M \epsilon \begin{pmatrix} H_2^5 \\ H_2^{45} \end{pmatrix},$$

$$M = \begin{pmatrix} -M_H + \frac{3}{2} \lambda_{H\Phi} v_{\Phi} & -\frac{15}{4\sqrt{6}} \overline{\lambda}_{HU} v_{\Phi} \\ -\frac{15}{4\sqrt{6}} \lambda_{HU} v_{\Phi} & M_U - \frac{19}{16} \lambda_{U\Phi} v_{\Phi} \end{pmatrix}.$$
(13)

Assuming an approximate equation for the elements of the mass matrix as  $M_{11}M_{22} - M_{12}M_{21} \sim M_W M_X$ , a pair of linear combinations of the SU(2)-doublet superfields have a small mass term of the order of  $M_W$ . Electroweak symmetry breaking is due to this pair of Higgs superfields, which are expressed by

$$H_1 = (C_1^{\dagger})_{11} H_1^{\overline{5}} + (C_1^{\dagger})_{12} H_1^{\overline{45}}, \tag{14}$$

$$H_2 = (C_2^{\dagger})_{11}H_2^5 + (C_2^{\dagger})_{12}H_2^{45}. \tag{15}$$

Here,  $C_1$  and  $C_2$  stand for the unitary matrices which diagonalize the mass matrix by  $(C_1)^T M C_2$ . The other pair of SU(2)-doublet superfields have naturally a large mass term of the order of  $M_X$ . Decoupling from theory below the GUT energy scale, these superfields do not affect flavor-changing neutral current processes at the electroweak energy scale nor energy evolutions of the gauge coupling constants for SU(3), SU(2), and U(1).

The masses of the down-type quarks and charged leptons can now be accommodated. At the GUT energy scale, the coefficient matrices appearing in Eq. (4) are given by  $\eta_d = -(C_1)_{11}\Gamma^{de}/\sqrt{2} - (C_1)_{21}\overline{\Gamma}^{de}/2\sqrt{3}$ ,  $\eta_e = -(C_1)_{11}(\Gamma^{de})^T/\sqrt{2} + 3(C_1)_{21}(\overline{\Gamma}^{de})^T/2\sqrt{3}$ , and  $\eta_u = 4(C_2)_{11}\Gamma^u - 4(C_2)_{21}\overline{\Gamma}^u/\sqrt{6}$ . Taking the generation basis in which  $\eta_d$  and  $\eta_e$  are diagonal, any values for the down-type quark and charged lepton masses are described in this model by imposing the conditions

$$\Gamma^{de} = -\frac{1}{2\sqrt{2}(C_1)_{11}} \left( 3\eta_d^D + \eta_e^D \right), \tag{16}$$

$$\overline{\Gamma}^{de} = -\frac{\sqrt{3}}{2(C_1)_{21}} \left( \eta_d^D - \eta_e^D \right). \tag{17}$$

The coefficients for the up-type quark masses should be taken as

$$\Gamma^{u} = \frac{1}{8(C_{2})_{11}} \{ (V_{CKM})^{T} \eta_{u}^{D} (U_{R}^{u})^{T} + U_{R}^{u} \eta_{u}^{D} V_{CKM} \},$$
(18)

$$\overline{\Gamma}^{u} = -\frac{\sqrt{6}}{8(C_{2})_{21}} \left\{ (V_{CKM})^{T} \eta_{u}^{D} (U_{R}^{u})^{T} - U_{R}^{u} \eta_{u}^{D} V_{CKM} \right\},$$
(19)

where  $U_R^u$  is an arbitrary unitary matrix. For the neutrinos, the condition on  $\Gamma^{\nu}$  is not altered and given by Eq. (9).

In summary, we have shown that the experimental results for the neutrinos are understood naturally in a GUT model based on SU(5) gauge group and N=1 supergravity. Only by incorporating symmetric 15 and  $\overline{\bf 15}$  representations for new Higgs superfields, the neutrinos can have, without fine-tuning, the masses which are observed. The CKM matrix and the MNS matrix become independent of each other. Their difference is considered a reasonable consequence of the fact that the relevant parameter values are freely given at the GUT energy scale. Inclusion of  $\bf 45$  and  $\bf \overline{45}$  representations is possible to accommodate the downtype quark and charged lepton masses.

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